Atmospheric circulation I

Reading: GPC Ch 6

Outline:

- Energy balance of the atmosphere
- Scales of motion
- Forces that determine the atmosphere and ocean circulation
    - Eulerian/Lagrangian descriptions of motion
    - Forces: pressure gradient, gravity, friction, Coriolis
    - Equations of motion
- Simplifications of the equations of motion: hydrostatic balance, geostrophic balance
- Thermal wind

The atmospheric general circulation (AGC) and climate

Atmospheric motions are generated by geographic variations in surface heating caused by meridional gardients of insolation, albedo variations, etc... By transporting energy, AGC winds generally act to offset the effects of these heating variations on atmospheric T. AGC transports heat from the tropics to the poles.

Energy balance of the atmosphere

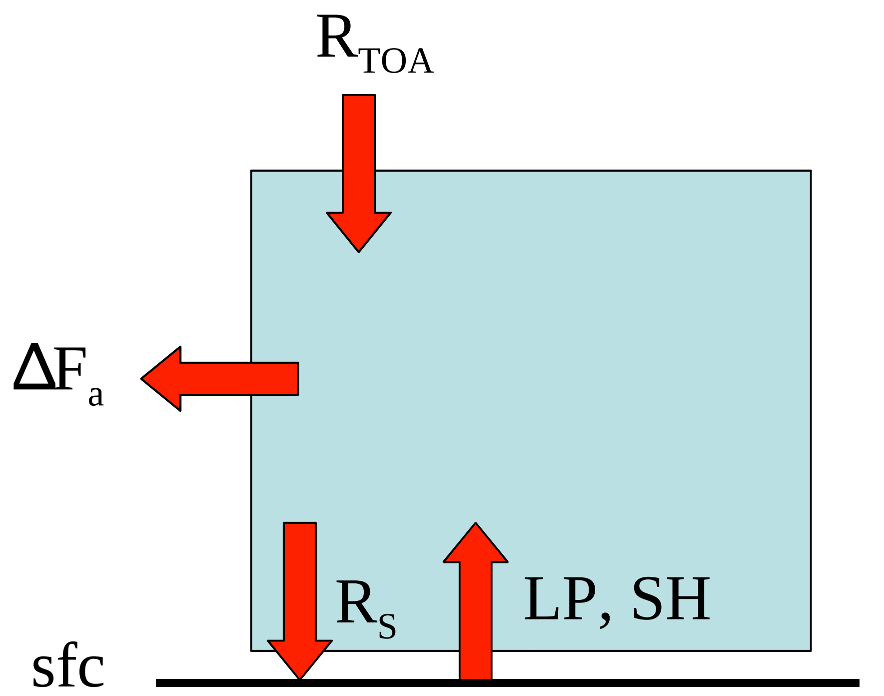
$$\frac{\partial E_a}{\partial t} = R_a + \mathrm{LP} + \mathrm{SH} - \Delta F_a$$

where

$$R_a = R_{\mathrm{TOA}} - R_s$$

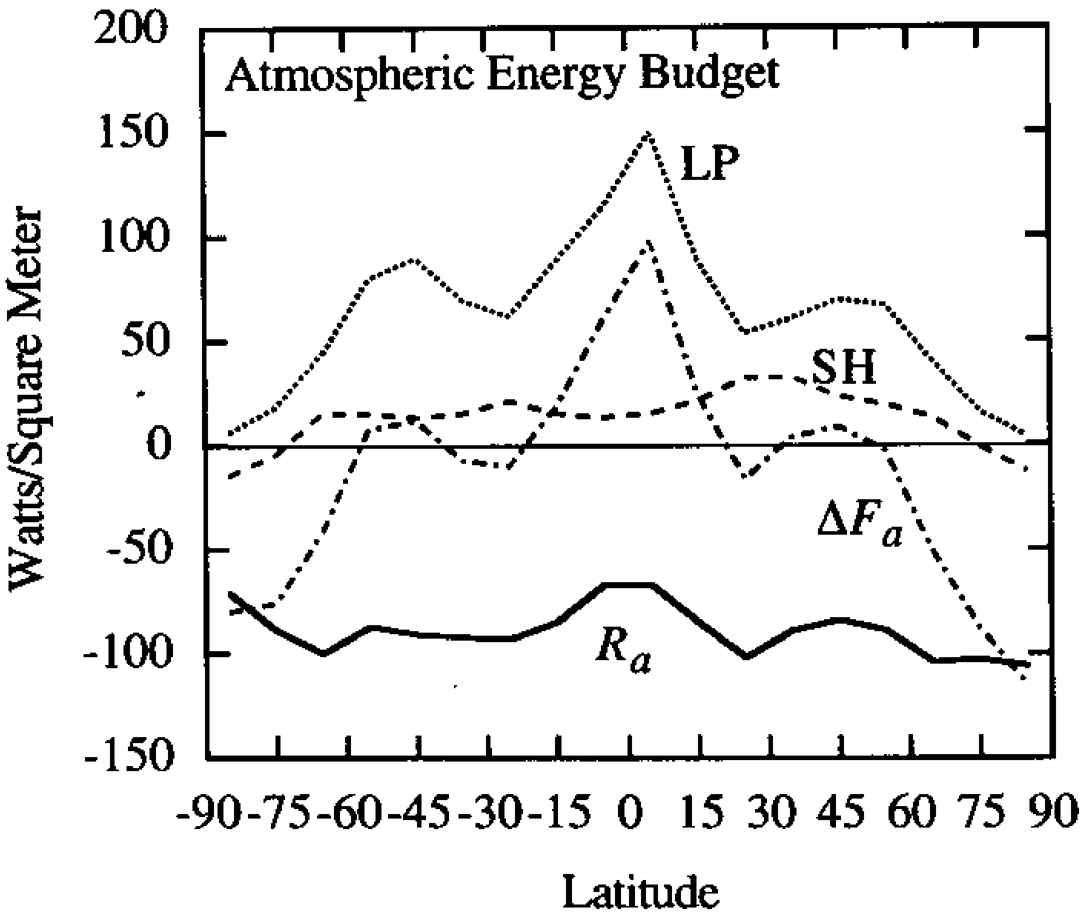
Energy balance

$$R_a + \mathrm{LP} + \mathrm{SH} = \Delta F_a$$



Local balance of an atmospheric column of unit horizontal area

Energy balance: $R_a = \Delta F_a - (LP + SH)$

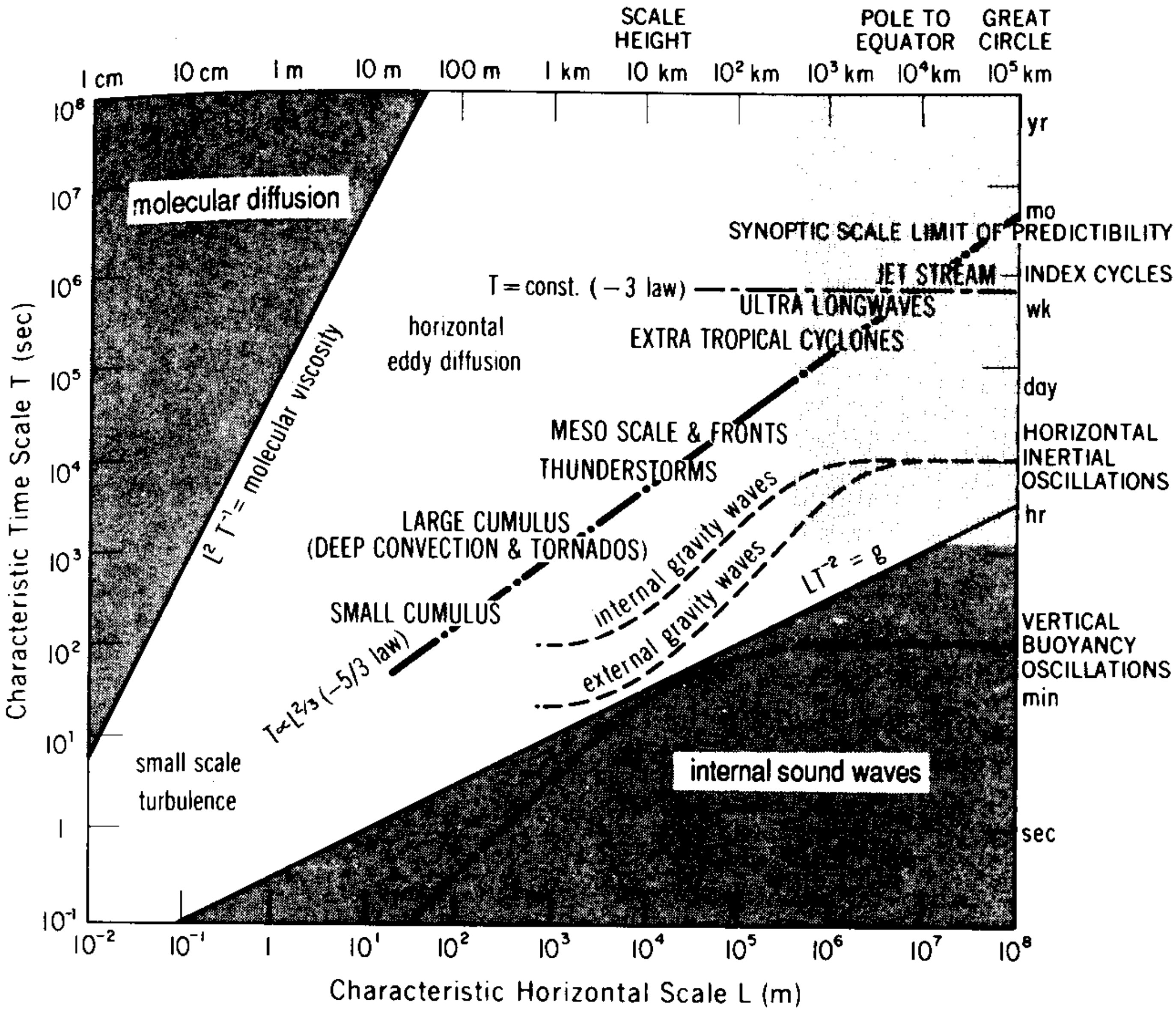


Average over longitude and over 1 y:

- LP latititude distribution corresponds to precipitation and is reflected in the distr. of $\Delta F_{a.}$
- SH is relatively small
- R_a is nearly indep. of latitude. Average cooling of ~90 W/m^2 (-1.5 ^{0}C/day) compensated by LP+SH where $\Delta F_{a.}$ is small (in the belt 20-60 degrees N and S).

Latent and sensible ***heats*** the atmosphere
Net radiation ***cools*** atmosphere
Atmosphere transports heat from tropics to polar regions (~100 W/m^2)

Atmospheric motions and the meridional trasport of energy

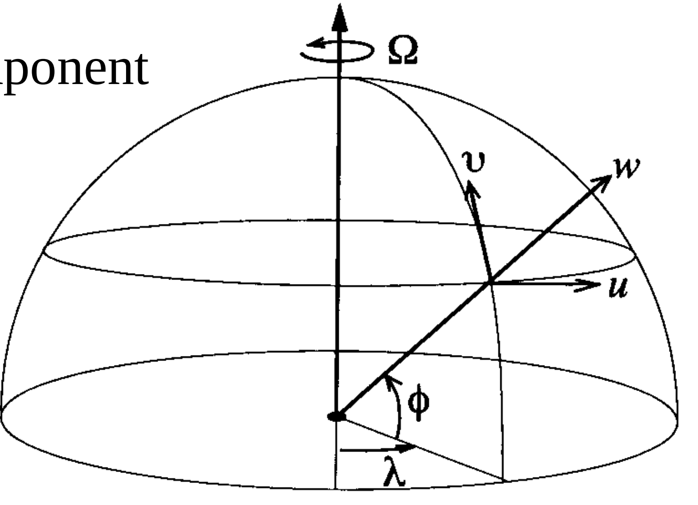


Wind components on a spherical earth

v(longitude, latitude, height) = **v**(λ,ϕ,z) = (u, v, w)

u - zonal wind speed (east is +ve)
v - meridional wind speed (north is +ve)
w - vertical wind speed (up is +ve)
ω - pressure velocity (the vertical component of velocity can be measured in terms of the rate of change of altitude, or the rate of change of pressure)

Local cartesian coordinates on a sphere of radius a



$u = a \cos\phi \, D\lambda/Dt$

$v = a \, D\phi/Dt$

$w = Dz/Dt$

$\omega = Dp/Dt$

where D/Dt = temporal tendency experienced by an air parcel moving with the flow.

In hydrostatic balance, $\omega \cong -\rho g w$

Forces that determine the atmospheric and ocean circulation

Reference: Holton, "An introduction to dynamic meteorology"

5. Fundamental governing law

NOTATION: vectors are boldface, 'normal' are scalars. The components of a vector **a** are denoted as ax, ay, az (so that **a** = (ax, ay, az)).
The exceptions are with position **x** = (x,y,z) and velocity **v** = (u, v, w), which are widely used notations.

Newton's second law

$$\mathbf{F} = m\,\mathbf{a} \qquad (1)$$

applies to a mass element.
BUT fluid is continuous, not discrete, so "mass of fluid" has no clear meaning.
Hence, we need to rewrite (1) in terms of *force per unit volume:* $\mathbf{F} = \rho\mathbf{a}$.
It is often easier to use *volume per unit mass* α instead of density ρ:
$\alpha = 1/\rho$ so that

$$\mathbf{a} = \alpha\mathbf{F} \qquad (2)$$

2. Eulerian and Lagrangian descriptions of motion

Usually we measure properties of a fluid as a function of fixed position and time ("***Eulerian description***"), e.g. $\mathbf{v}(\mathbf{x},t)$ or $T(\mathbf{x},t)$.

But note that if you measure a property (temperature, say) of a fluid at a fixed point $\mathbf{x}$ at two different times t_1 and t_2, you don't actually measure the same parcel of fluid, since it is (in general) moving.

This is particularly problematic if you were going to apply Newton's second law, since description of acceleration $d\mathbf{v}/dt$ involves the velocity of the *same* parcel at two different times (even if the gap between the times is infinitesimally small).

In other words, if you measure the velocity of the fluid in the same position $\mathbf{x}$ at two different times, $\mathbf{v}(\mathbf{x},t)$ and $\mathbf{v}(\mathbf{x},t+dt)$, the acceleration ***is not*** $(\mathbf{v}(\mathbf{x},t+dt)-v(\mathbf{x},t)) / dt$.

The solution is to describe fluid properties as if you were to *follow the fluid parcel* ("***Lagrangian description***")

NOTATION:

D/Dt is the time derivative following the fluid parcel
So: Lagrangian acceleration is $D(u,v,w)/Dt$
and we can write Newton's second law for a fluid parcel as

$$\mathbf{D(u,v,w)/Dt} = \alpha\Sigma_{all\_forces}\,\mathbf{(Fx,Fy\ ,Fz\ )} \qquad (3)$$

Here the net force has been written as the vector sum of all forces on the fluid.

3. Forces on the fluid that are important in the atmosphere and in the ocean:

- ***pressure force;***
- ***gravity;***
- ***frictional force;***
- ***Coriolis force*.**

Form of these forces:

- $\mathbf{F^{pressure} = (F^{pressure}_x, F^{pressure}_y, F^{pressure}_z) = (-\partial p/\partial x, -\partial p/\partial y, -\partial p/\partial z)}$ (4)

- $\mathbf{F^{gravity} = (0, 0, -\rho g)}$ **where** $\mathbf{g = 9.8 m/s^2}$ (5)

- $\mathbf{F^{friction} = (F^{friction}_x, F^{friction}_y, F^{friction}_z)}$ (6)

  Friction usually has a complex dependence.
  A simple way to model it is to assume that it is proportional to the parcel's velocity:

$\mathbf{F^{friction}} = -k\,\mathbf{v} = -(ku, kv, kw)$ (7)
where k is the friction coefficient.

This type of friction parameterization is known as *Rayleigh* friction.

• The Coriolis 'force' is strictly not a force – it arises because our usual frame of reference on the earth – longitude, latitude, height – is actually a non-inertial frame of reference since the earth is rotating (and hence the frame of reference is accelerating). The Coriolis force is here to compensate for the effects of using an accelerating frame of reference.

The Coriolis force for atmosphere/ocean is well approximated by this expression:

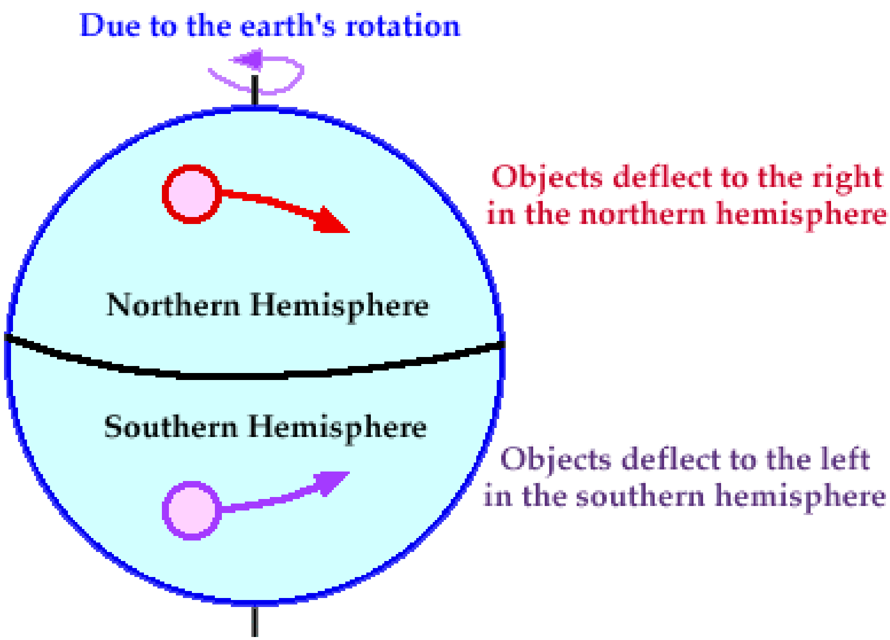
$\mathbf{F}^{coriolis} = (\rho f v, -\rho f u, 0)$ (8)

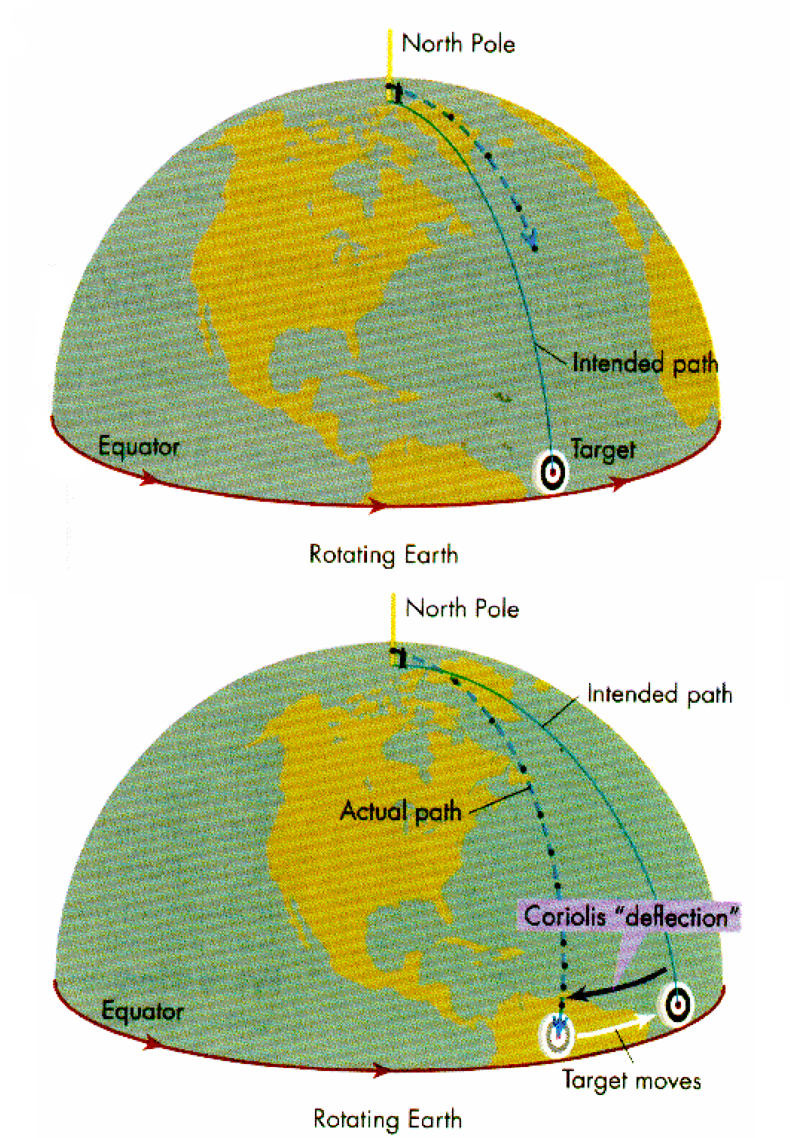
where f is the Coriolis parameter $\mathbf{f = 2\ \Omega \sin}\ \phi$ (9)

Ω is the rotation rate of the earth and ϕ is the latitude.

What it means is that there will be a force on the parcel perpendicular to the direction of the parcel motion (to the ***right*** in the northern hemisphere, to the ***left*** in the southern hemisphere), and with a magnitude proportional to the speed of the parcel.

A 3-d scketch of the Coriolis force





4. Putting everything together

With all the forces involved, we now have Newton's second law for a geophysical fluid ('geophysical' since you have to account for the earth's rotation):

$$\frac{Dv}{Dt} = \alpha\left(F^{pressure} + F^{gravity} + F^{coriolis} + F^{friction}\right)$$

writing it out fully and in its components:

$$\begin{aligned}
\frac{Du}{Dt} &= -\alpha\frac{\partial p}{\partial x} + fv + \alpha F_x^{friction} && zonal_component \\
\frac{Dv}{Dt} &= -\alpha\frac{\partial p}{\partial y} - fu + \alpha F_y^{friction} && meridional_component \qquad (10) \\
\frac{Dw}{Dt} &= -\alpha\frac{\partial p}{\partial z} - g + \alpha F_z^{friction} && vertical_component
\end{aligned}$$

5. Simplifications of the equations of motion

•Hydrostatic balance

Since the 'horizontal' extent of the atmosphere is so much larger than the vertical extent (thousands of kilometers as opposed to kilometers), large-scale vertical motions w can be considered much smaller than u or v. It also means that Dw/Dt is also small. If we also assume friction is negligible in the vertical ($F^{friction}_z \sim 0$), then we can throw those terms our from the vertical component of (10). If we do that, note that we just get back our old friend the hydrostatic balance: $\alpha \frac{\partial p}{\partial z} = -g \quad or \quad \frac{\partial p}{\partial z} = -\rho g$ (11)

- **Geostrophic balance**

If we concern ourselves only with atmospheric or oceanic motions that change slowly (over a number of days) over time, we can simplify the horizontal equations of motion in this way. Lets say that U is a characteristic wind speed, then the scale of the acceleration term is

$$Du/Dt \sim Dv/Dt \sim U / (\text{several days})$$

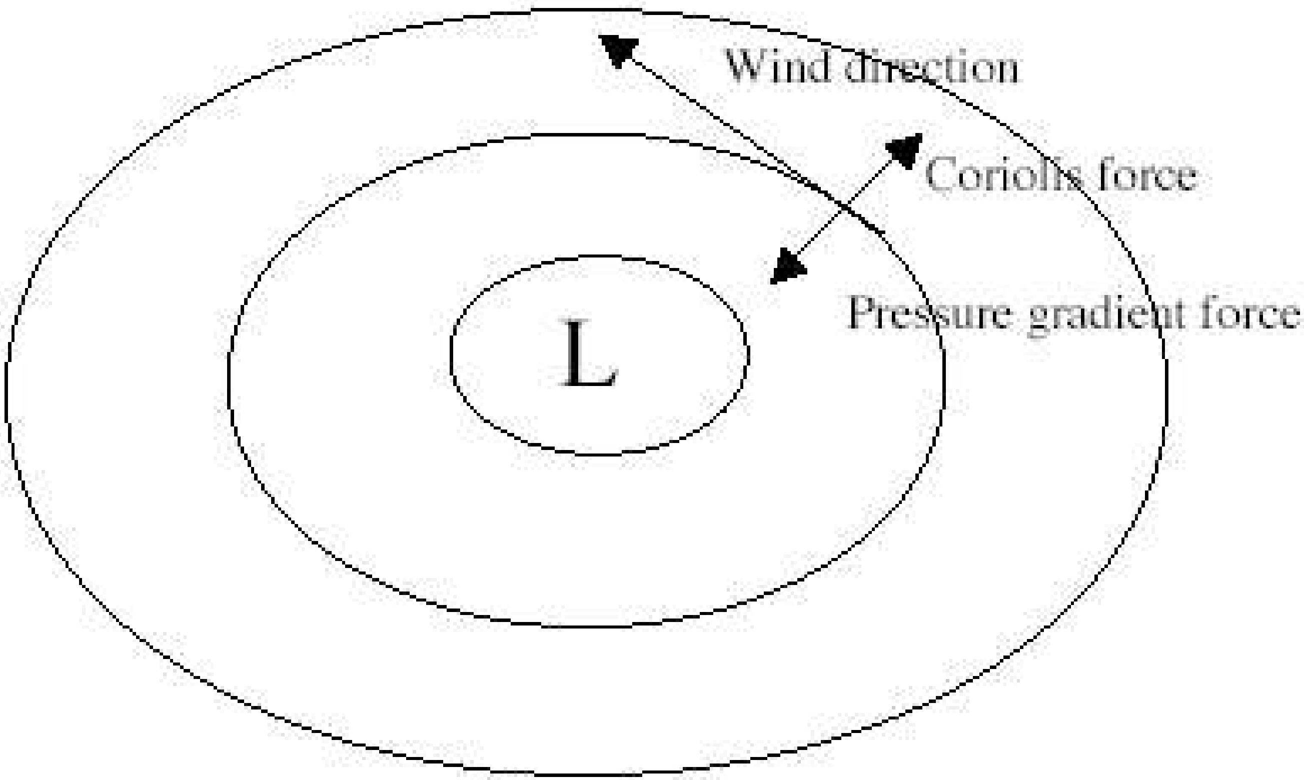
Note that a similar analysis for the Coriolis terms yield

$$fv \sim fu \sim U/(1\ \text{day}).$$

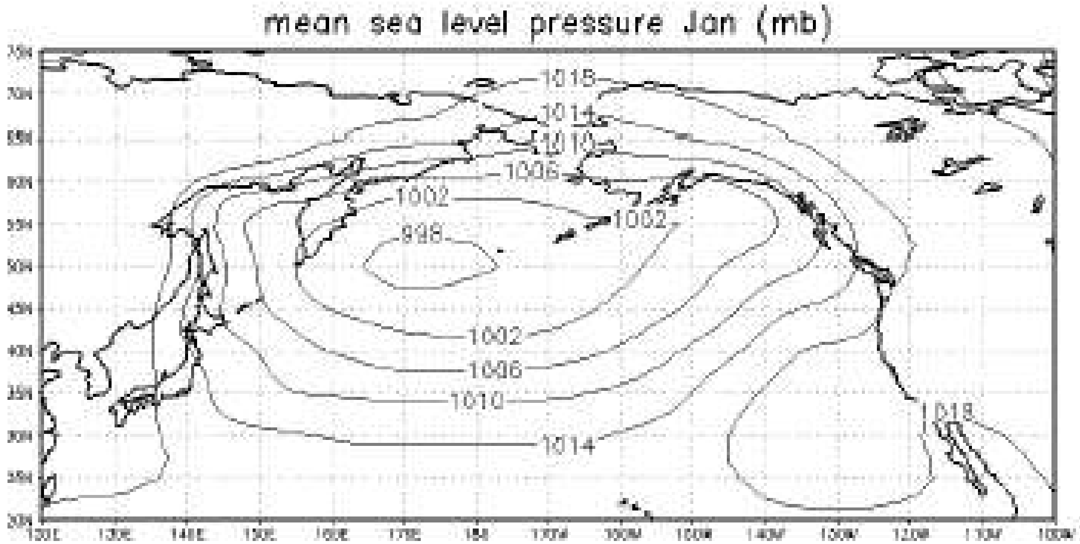
In other words, for motions that change slowly with time, the acceleration term in (10) are negligible compared to the Coriolis term. A similar "scale analysis" will show this is also true when compared to the pressure gradient term. Furthermore, if we look only at motions above the boundary layer, then the frictional term is also small, and we can rewrite the horizontal equations of motion in (10) as:

$$\begin{aligned} fv &= \alpha \frac{\partial p}{\partial x} && zonal\_component \\ fu &= -\alpha \frac{\partial p}{\partial y} && meridional\_component \end{aligned} \qquad (12)$$

This is called the ***geostrophic balance***, and the motions are called geostrophic motions. A defining characteristic of geostrophic motions is that the wind direction is parallel to contours of pressure. In ***the northern hemisphere, the pressure gradient force is always to the left of the wind direction*** (as f changes sign, the pressure gradient force is to the left of the wind direction in the southern hemisphere). Note that this is opposite of the Coriolis force, which acts to the right of the wind direction. In other words, the pressure gradient force balances the Coriolis force so that the net force is zero – hence, geostrophic ***balance***.



Example: let's compute the geostrophic zonal velocity at 180°W, 37°N, given the surface pressure map below:



At 180°W, 30°-45°N $\quad\quad$ $\Delta p \sim 14\text{mb} = 1400\text{Pa}$ (from above)

$\Delta y \sim 15° \times 2\pi/180° \times$ radius of earth ($\sim 6.4\times10^6$m) $= 1.7\times10^6$m

$f = 2\Omega \sin\phi \quad \sim \quad 2 \times 0.7\times10^{-4}\ s^{-1} \times \sin(37°) = 0.84\times10^{-4}\ s^{-1}$

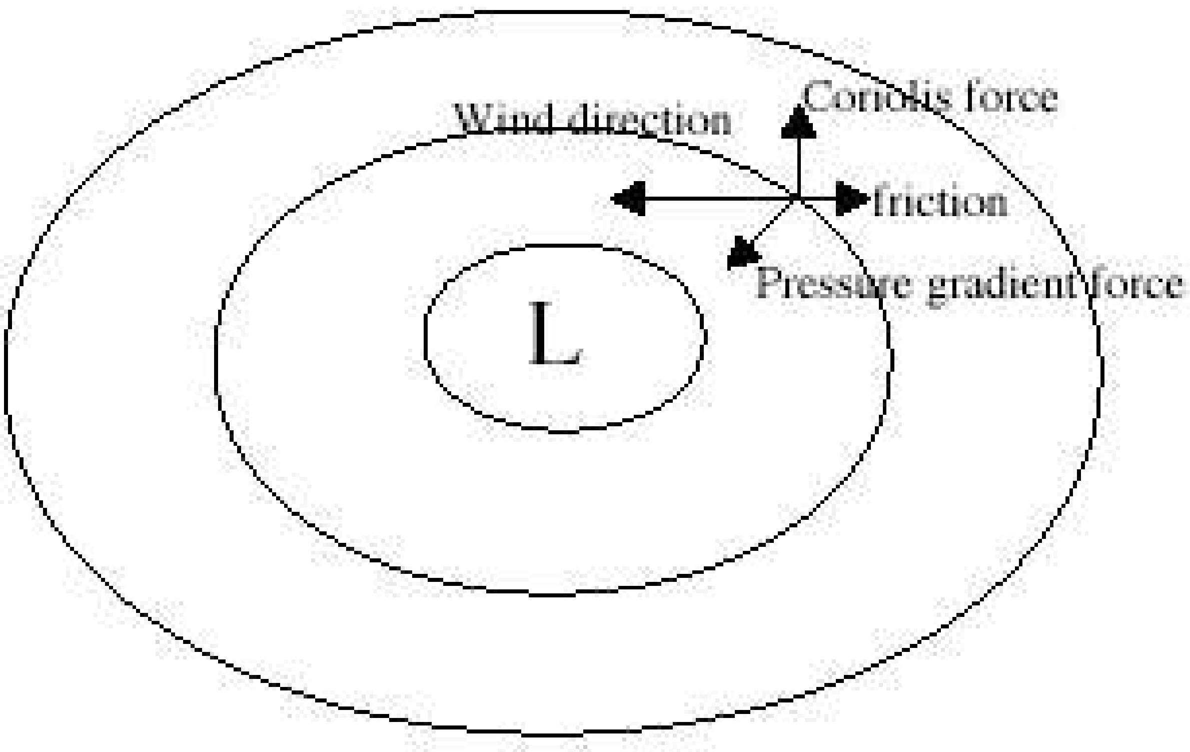
Hence the geostrophic zonal wind speed at 180°W, 37°N is:

$$
\begin{aligned}
u &= -(1/f)\,(1/\rho)\,\partial p/\partial y \\
&= -(1/0.84\times10^{-4})\,(1/1.3\text{kg m}^{-3})\,(1400\text{ Pa}/\ 1.7\times10^{6}\text{m}) \\
&= 7.5\text{ m/s}
\end{aligned}
$$

Note: pressure force is northward, and wind is westerly (eastward)

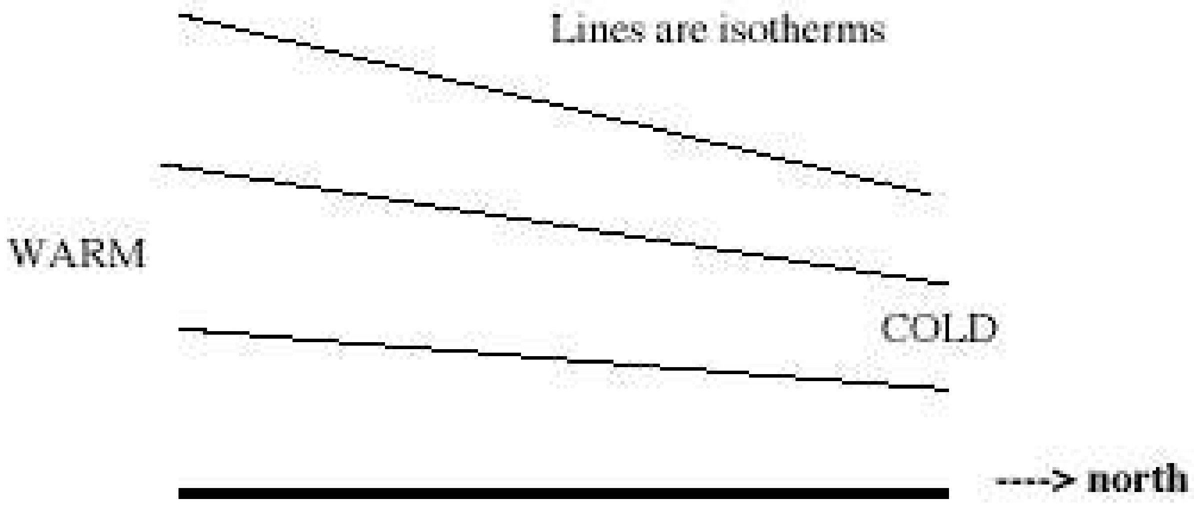
- **Friction and flow across isobars**

Note that in the boundary layer, friction is usually not negligible compared to the terms in the geostrophic balance. As a consequence, friction will slow the flow, the Coriolis force weakens as a result, and the wind direction will turn slightly inwards into a low pressure region from its original flow parallel to the pressure contours. The balance in this flow is a 3-way balance between the frictional force, Coriolis, and pressure gradient. This effect is strongest when f is small (i.e. low latitudes):



6. Thermal wind equation

It turns out that the geostrophic relationship, when combined with the hydrostatic relationship, can be used to describe zonal flows associated with meridional gradients in pressure (caused by, say, equator-to-higher latitude temperature gradients).



The analysis is easier to do in pressure rather than height as the vertical co-ordinate. Note that if we use constant pressure surfaces as our 'horizontal', then gradients of geopotential height (denoted Φ - it is equal to gz, or the actual height multiplied by the gravitational acceleration) take the place of the pressure gradients.

Meridional component of the geostrophic balance (in pressure co-ordinates):

$$f\,u = -\ \partial\Phi/\partial y \qquad (13)$$

Hydrostatic equation:

$$\partial p/\partial z = -\rho\, g \quad \Rightarrow \quad g\partial z/\partial p = -\ \alpha$$
$$\Rightarrow \quad \partial\Phi/\partial p = -\ \alpha \qquad (14)$$

Differentiate (14) with respect to y to get $\partial^2\Phi/\partial y\partial p = -\ \partial\alpha/\partial y$ (15)

Substitute (13) into (15) to eliminate Φ (assume f can be treated constant):

$$f\ \partial u/\partial p = \partial\alpha/\partial y \qquad (16)$$

But: from the equation of state, $p = \rho RT$ or $\alpha = 1/\rho = RT/p$ so that (16) can be written:

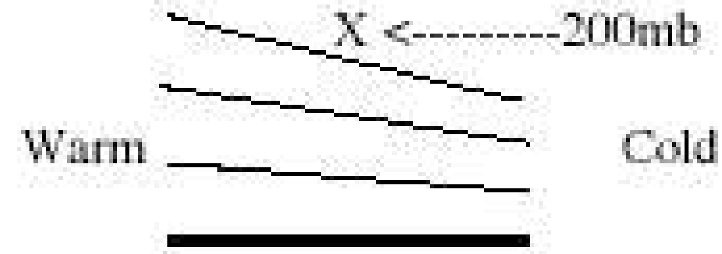
$$f\ \partial u/\partial p = (R/p)\partial T/\partial y \qquad \text{OR}$$

$$\frac{\partial u}{\partial \ln p} = \frac{R}{f}\frac{\partial T}{\partial y}$$ ***Thermal wind relation*** (17)

The upshot of this relationship is that the vertical shear of the zonal wind is directly related to the meridional gradient in temperature, through this relationship.

Example: let's estimate the zonal wind speeds in the subtropical/midlatitude jets for the northern winter given that the meridional temperature drop is ~30K over 6000km.

$\partial T/\partial y \sim -30K/6x10^6 m$



R = 287 J/(kg K)

$f = 2\Omega \sin \phi$

$\sim \Omega = 0.7x10^{-4} s^{-1}$ (sin ϕ is around 0.5 in the midlatitudes)

Let's assume u at the surface is small, and that we are looking at winds at 200mb:

Δu = u (1000mb) – u(200mb) = - u(200mb)

Δlnp = ln(1000mb) – ln(200mb) = ln(1000/200) = ln (5)

Hence $\frac{\partial u}{\partial \ln p} = \frac{R}{f}\frac{\partial T}{\partial y}$ => u(200mb) $= [(287/0.7x10^{-4}) \text{ x } (30/6x10^6)] \text{ x } \ln(5)$

$= 32.8$ m/s