Radiative Transfer and Climate II

Reading: GPC Ch3.

Outline:

- Absorption and emission of radiation, Lambert-Bouguet-Beer law
- Integral equation of transfer
- Simple flux form of the radiative transfer equation as applied to terrestrial radiation
- Radiative equilibrium: conceptual model and detailed calculations
- Radiative and radiative-convective equilibrium temperature profiles

Absorption of radiation (simplified treatment)

Simple case: ***no absorption***

Distance s →

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Intensity F remains the same

With absorption

Distance s →

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Intensity F decreases. The ***Lambert-Bouguet-Beer*** law describes this behavior:

$$\frac{dF}{ds} = -k\rho_a F \qquad (3.12)$$

k is the absorption coefficient (units: m^2/kg)
ρ_a is the density of the absorber (units: kg/m^3)
F is the intensity of radiation

If we assume k and ρ_a are constant, we can solve for (3.12) for F:

$$F(s) = F(s=0)e^{-k\rho_a s} \qquad (A)$$

Optical depth, mean free path

Let's introduce a useful simplification:

$$\tau = k\rho_a s$$

τ is the optical depth

Then equation (A) can be written as

$$F(\tau) = F(\tau=0)e^{-\tau} \qquad (3.17)$$

What does optical depth mean? It is a measure of 'optical' distance, done in such a way that 1 unit of τ means the intensity F decreases by a factor 1/e. It has ***no units***.

The ***mean free path*** is the actual distance travelled for one optical depth:
Mean free path = $1/k\rho_a$

The same equation is obtained from more realistic calculations.

Emission and absorption

Let's now let the matter emit as well:

Distance s →

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here F increases since medium emits here F reaches constant, F=B

$$\frac{dF}{ds} = -k\rho_a F + k\rho_a B \qquad (3.29)$$

addition al term for emission – B is the Planck function

Note the following:

- There is no scattering in this model – just absorption and emission
- If F=B, then the intensity F remains constant. In other words, the radiation absorbed by the medium is balanced by the radiation emitted by the medium.
- The absorption constant k is generally a function of frequency ν, as is B. So F is generally a function of the frequency.
- The special case of k independent of frequency is called a gray case
- B=0 is a good approximation for solar radiation in the atmosphere (why?) – but scattering is neglected
- If B is included, it gives a decent description of the terrestrial (LW) radiation. Scattering is weak at the LW frequencies.

Integral equation of transfer

Let's now solve for the equation (3.29) for the case of the atmosphere.
Assume:

- The path of the radiation is normal to the surface – so going straight
- The temperature T, and therefore B, is only a function of height (plane-parallel approximation)

Convention: τ=0 at surface, so τ increases with height

(3.29) can be written as:

$$\frac{dF}{d\tau} = -F + B \qquad \text{(B)}$$

Solve (B) by using the integrating factor $e^{-\tau}$:

$$\frac{dF}{d\tau} + F = B$$

so
$$\frac{d(e^{\tau} F)}{d\tau} = Be^{\tau}$$

Integrate from τ=0 (surface) to $\tau(z)$ (optical depth at height z). Recall that B is a function of temperature T, and that temperature varies with height:

$$e^{\tau(z)}\, F(\tau(z)) - e^{0}F(\tau{=}0) = \int_0^{\tau(z)} B(T)\, e^{\tau}\, d\tau$$

or
$$F^{\Uparrow}(\tau(z)) = F^{\Uparrow}(\tau{=}0)\, e^{-\tau(z)} + \int_0^{\tau(z)} B(T)\, e^{\tau-\tau(z)}\, d\tau \quad (3.34)$$

(3.34) gives the ***upward*** flux at height z, given the flux at the surface (hence $F^{\Uparrow}$). The first term on the RHS is contribution from the surface but with an attenuation $e^{-\tau(z)}$ because of the absorbers in between the surface and height z, whereas the latter comes from contribution from layers in the atmosphere between the surface and $\tau(z)$, again with attenuation (but this time $e^{\tau-\tau(z)}$ – can you see why it is of this form?)

By the same token you can work out the ***downward*** flux at height z by adding up all contributions from the top of atmosphere (TOA) to level z:

$$F^{\Downarrow} = F^{\Downarrow}(\tau(TOA))\, e^{\tau(z)-\tau(TOA)} + \int_{\tau(z)}^{\tau(TOA)} B(T)\, e^{\tau(z)-\tau}\, d\tau \qquad (C)$$

Note that for terrestrial radiation, the first term on the RHS of the above equation is zero (since there is no incoming flux of terrestrial radiation at the top of the atmosphere),

So, the **net flux** at level z is given by

$$F(z) = F^{\Uparrow}(z) - F^{\Downarrow}(z)$$

The importance of the flux to climate is how the radiation heats up the atmosphere. The expression for this is:

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_p}\frac{\partial F}{\partial z} \qquad (3.38)$$

where ρ is the air density and c_p is its specific heat capacity.

Simple flux form of the radiative transfer equation as applied to terrestrial radiation

If we ignore the frequency dependence of B over the terrestrial frequency band, we can come up with an approximate solution for the outgoing terrestrial radiation at the top of the atmosphere by integrating (3.34) over the all frequencies to get:

$$F^{\Uparrow}(TOA) = \sigma T_s^4 \, e^{-\tau(TOA)} + \int_0^{\tau(TOA)} \sigma T^4 \, e^{\tau-\tau(TOA)} \, d\tau \qquad (3.39)$$

Likewise, we can also derive the downward flux at the surface from equation (C):

$$F^{\Downarrow}(z=0) = \int_0^{\tau(TOA)} \sigma T^4 \, e^{-\tau} \, d\tau \qquad (3.40)$$

The important thing here to understand for (3.39) and (3.40) is which part of the integration contributes to the $F^{\Uparrow}(TOA)$ or $F^{\Downarrow}$ at the surface. Of course, every part of the atmospheric column contributes, but because of the absorption by the atmosphere, only the portion of the atmosphere within ~1 optical depth from the top of the atmosphere (in the case of $F^{\Uparrow}(TOA)$) or the surface (in the case of $F^{\Downarrow}$ at the surface) *effectively* contributes.

Radiative equilibrium - a conceptual model

2 atmospheric layers at 0.5km and 2km above surface, blackbody to LW, transparent to SW. What will the temperature of the two layers be?

Solve it the same way as before: compute energy balance for each layer and the surface

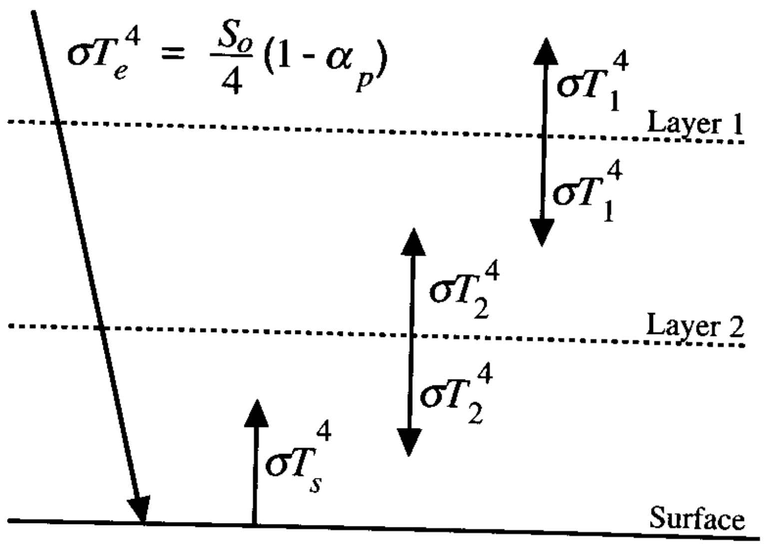


Fig. 3.10 Diagram of simple two-layer radiative equilibrium model for the atmosphere–Earth system, showing the fluxes of radiant energy.

$$\frac{S_0}{4}\left(1-\alpha_p\right)=\sigma T_e^4=\sigma T_1^4$$ ← TOA energy balance

$$\sigma T_2^4=2\sigma T_1^4$$ ← Layer 1 energy balance

$$\sigma T_1^4+\sigma T_s^4=2\sigma T_2^4$$ ← Layer 2 energy balance

$$\frac{S_0}{4}\left(1-\alpha_p\right)+\sigma T_2^4=\sigma T_s^4$$ ← Surface energy balance

Solution: $$T_s^4=3\frac{\left(S_0/4\right)\left(1-\alpha_p\right)}{\sigma}=3T_e^4$$

Recall for single-layer case: $T_s^4 = 2\ T_e^4$

Solution: $T_s^4 = 3\, T_e^4$ with two layers

Values:

T_s=335.6 K=62.5 ^{0}C T_1=T_e=255 K=-18 ^{0}C T_2=303.2 K=30 ^{0}C

By extension, if such a model has an arbitrary number n of layers, $T_s^4=(n+1)T_e^4$

We now add a thin layer of atmosphere of emissivity ε at the TOA (~ stratosphere) absorbing no solar radiation:

$\varepsilon\sigma T_e^4=\varepsilon 2\sigma T_{strat}^4$ (absorption from below=emission up and down).

We also add a thin layer of atmosphere of emissivity ε and temperature T_{SA} near the surface, absorbing no solar radiation:

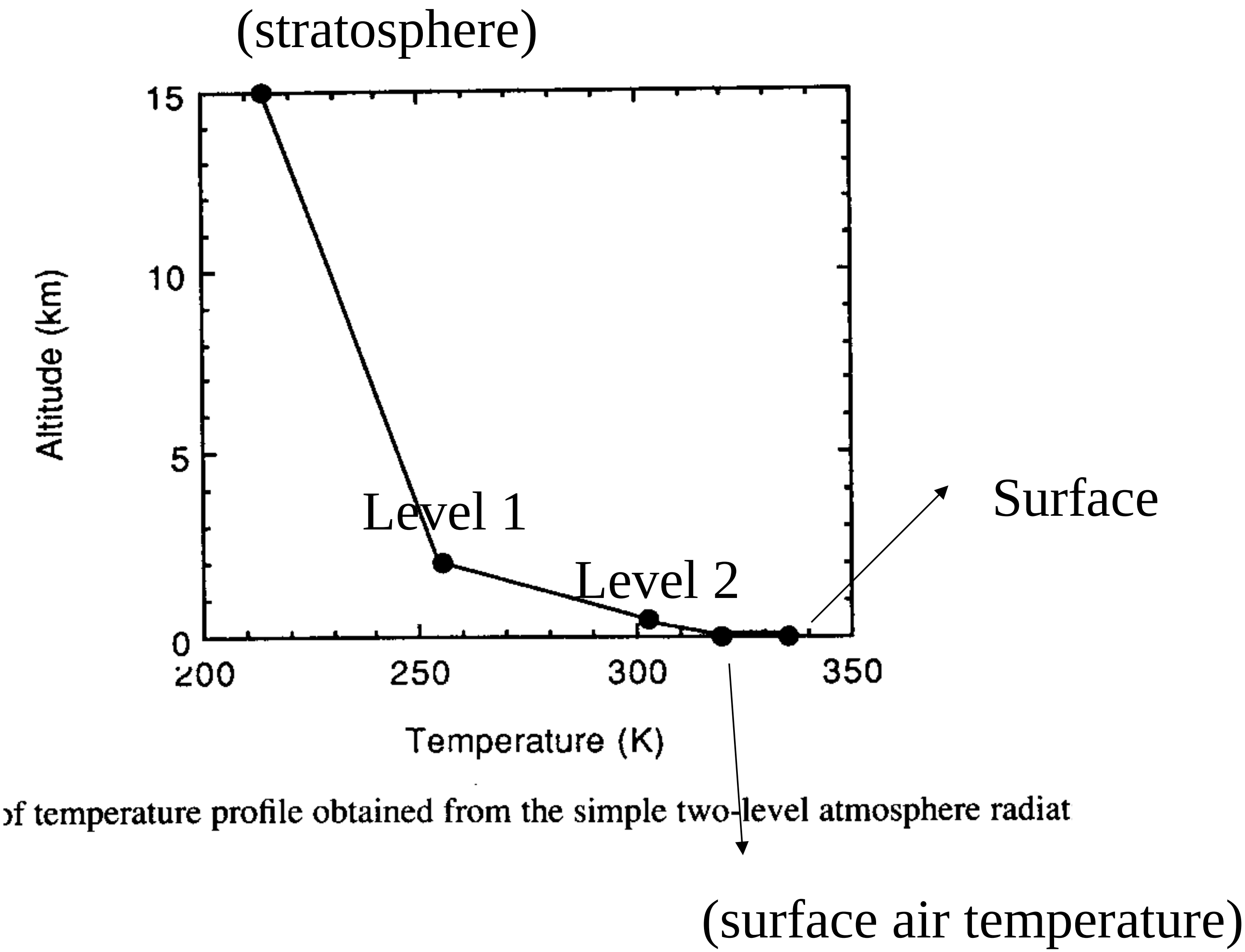
$\varepsilon\sigma T_s^4+ \varepsilon\sigma T_2^4 =\varepsilon 2\sigma T_{SA}^4$ (absorption from surface and layer above=emission up and down).

Solution: $T_{SA}^4=(T_s^4+T_2^4)/2$ $T_{strat}^4=T_e^4/2$

Values: $T_{SA}=320\text{ K}=47\ ^0\text{C}$ $T_{strat}=214\text{ K}=-59\ ^0\text{C}$

In pure radiative equilibrium T_s and T_{SA} are different.
This discontinuity is caused by the absorption of solar radiation *at* the surface and is usually greatly suppressed in reality because of efficient heat transport by conduction and convection.

Radiative equilibrium is not a good approximation for T_s which turns out to be much hotter than observed, because latent and sensible heat fluxes remove substantial amounts of energy from the surface.


(stratosphere)
15
10
5
0
Altitude (km)
200
250
300
350
Temperature (K)
Level 1
Level 2
Surface
of temperature profile obtained from the simple two-level atmosphere radiat
(surface air temperature)

Radiative and radiative-convective equilibrium temperature profiles

Radiative equilibrium temperature profile:

• radiative energy balance achieved at all levels
• no energy transport by atmospheric motions
• no latent and sensible heat fluxes

To improve the model, one can solve he radiative transfer equation for global mean terrestrial conditions (because horizontal transport of energy by atmospheric motions affects the local climate).

This involves construction of appropriate models of the transmission of the various frequency bands of importance in the atmosphere, insertion of these into a computational implementation of the radiative transfer equation and iteration to obtain a steady solution.

In the global mean model all variables depend only on altitude.
Globally average insolation and solar zenith angle are used.

The following need to be specified:

- Water content - concentrated at lower altitudes
- CO_2 - generally well-mixed in latitude and height
- Ozone - Important in stratosphere
- Aerosols - affects transmission of SW and LW;
  sulfate aerosols
- Surface albedo
- Clouds - we'll cover this in a bit….

However, there is one more problem…..

In the troposphere, radiative equilibrium temperature profiles are hydrostatically unstable. In the real atmosphere, atmospheric motions move heat away from the surface and mix it through the troposphere. In the sketch of global flux energy balance we have seen that the energy removal from the surface by the transport of heat and water vapor by air motions is

29:50=x:100 x=58%

and that by net LW emission is

21:50=x:100 x=42%.

The global mean temperature profile is not in radiative equilibrium but in radiative-convective equilibrium.

The vertical flux of energy by atmospheric motions must be included in the model.

The simplest way to do so is "*convective adjustment*".

The lapse rate is not allowed to exceed a critical value, e.g. 6.5 K/Km . Radiative processes would make it greater; so it is assumed that non-radiative upward heat transport occurs, that mantains Γ below the threshold while conserving energy.
This artificial vertical redistribution of energy is meant to represent the effect of atmospheric motions without explicitely calculating non-radiative fluxes and air motions.
This "*adjusted layer*" extends from the surface to the tropopause.
No a priori reason exists for choosing Γ = 6.5 K/Km, except other than the fact that the resulting profile turns out to be very close to the observed global mean value.
To better understand all this, we will have to discuss the lapse rate in the atmosphere and the concepts of potential temperature and static stability.

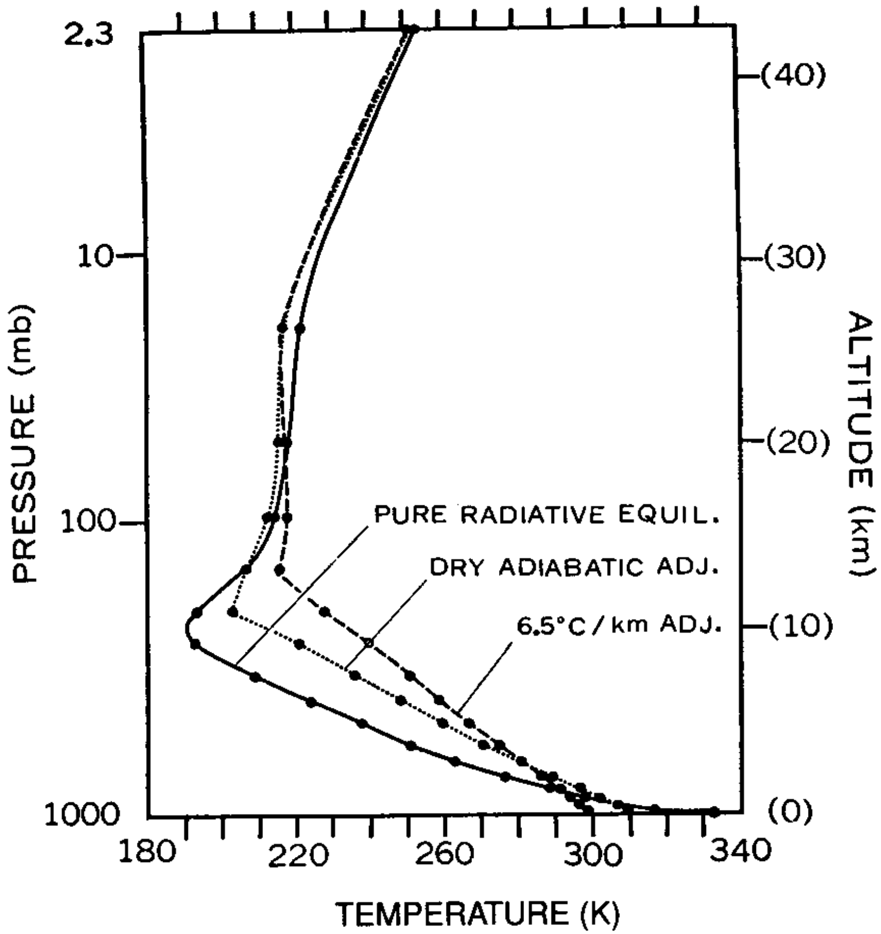


Fig. 3.16 Calculated temperature profiles for radiative equilibrium, and thermal equilibrium with lapse rates of 9.8°C km^{-1} and 6.5°C km^{-1}. [From Manabe and Strickler (1964). Reprinted with permission from the American Meteorological Society.]

Dry adiabatic equilibrium: Γ=9.8 K/km

Mean Γ in the troposphere from radiosondes: Γ=6-7 K/km

Radiative equilibrium is too cold in the troposphere and too warm at the surface.